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$\therefore h = 37^\circ 37'$ or $137^\circ 18' 12''$. $h = 2$ hours, 30 minutes, 28 seconds, or 9 hours, 9 minutes, 12.8 seconds.

\therefore sidereal time $= 1$ hour, 22 minutes, 28 seconds, or 8 hours, 1 minute, 12.8 seconds.

36. Proposed by J. K. ELLWOOD, A. M., Principal of the Colfax School, Pittsburg, Pennsylvania.

"What is the length of a chord cutting off one-fifth of the area of a circle whose diameter is 10 feet?"

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

Let the chord subtend an angle $= 2\theta$, $a =$ radius of circle. Then the length of the chord $= 2a \sin \theta$.

$$\therefore a^2 (\theta - \sin \theta \cos \theta) = \frac{1}{5} \pi a^2.$$

$$\therefore \theta - \sin \theta \cos \theta = \frac{1}{5} \pi, \therefore \theta = 60^\circ 32' \text{ nearly.}$$

$$\therefore \text{chord} = 2a \sin \theta = 10 \sin \theta = 8.7064 \text{ feet.}$$

II. Solution by BENJ. F. YANNEY, A. M., Professor of Mathematics, Mount Union College, Alliance, Ohio, and Prof. P. S. BERG, Larimore, North Dakota.

Let $\theta =$ the angle at the center, subtended by the required chord. Then $10 \sin \theta =$ the length of the required chord. Now $\frac{2\theta}{360} \pi 25$, the area of the sector, $- 5 \sin \theta \sqrt{(25 - 25 \sin^2 \theta)}$, the area of the triangle, $= 5\pi$, the given area of the segment. Whence, by reduction, $\frac{\theta}{180} \pi - \sin \theta \cos \theta = \frac{\pi}{5}$.

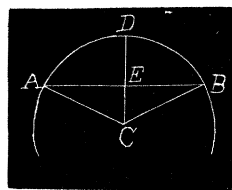
$$\therefore \frac{\theta}{90} \pi - 2 \sin \theta \cos \theta = \frac{2}{5} \pi. \therefore .0349065 \theta - \sin 2\theta = 1.256637.$$

From which we readily find, by supposition, the value of θ ; and from this, the value of $10 \sin \theta$ to be 8.706, the length of the chord required.

III. Solution by A. H. BELL, Hillsboro, Illinois.

By Reversion of Series. Let the given diameter $= 10 = D$ and $1/5$ of circle $= a\pi r^2/d$, radius $= r$. To obtain the greatest convergency in the series, let ACB , the angle at the center $= 2\theta$ and take the sector $ACD = r^2 \theta / 2$ and $r^2 \sin \theta \cos \theta / 2 = ACE$.

Then $r^2 (\theta - \sin \theta \cos \theta) / 2 = a\pi r^2 / 2d$ or $\text{arc} \theta = a\pi / d + \cos \theta \sqrt{(1 - \cos^2 \theta)} \dots \dots \dots (1)$.



Make $\cos \theta = x$, and when expanded,

$$\theta = \frac{a\pi}{d} + x - \frac{x^3}{2} - \frac{x^5}{2.4} - \frac{3x^7}{2.4.6} - \frac{3.5x^9}{2.4.6.8}, \text{ etc., } \dots \dots \dots (2).$$

By trigonometry or calculus, we have,

$$\text{arc} \theta = \frac{\pi}{2} - x - \frac{x^3}{2.3} - \frac{3x^5}{2.4.5} - \frac{3.5x^7}{2.4.6.7} - \frac{3.5.7x^9}{2.4.6.8.9}, \text{ etc., } \dots \dots \dots (3).$$

(2)–(3) and \div by 2, etc.,

$$y = \frac{(d-2a)\pi}{4d} = x - \frac{x^3}{6} - \frac{x^5}{40} - \frac{x^7}{112} - \frac{5x^9}{1152} - \text{etc.,} \dots \dots \dots (4).$$

$$\text{Assume } x = Ay + By^3 + Cy^5 + Dy^7 + Ey^9 + \text{etc.,} \dots \dots \dots (5).$$

The powers of x substituted in (4), $y = Ay +$

$$\left(B - \frac{A^3}{6}\right)y^3 + \left(C - \frac{A^2B}{2} - \frac{A^5}{40}\right)y^5 + \left(D - \frac{A^2C}{2} - \frac{AB^2}{2} - \frac{A^4B}{8} - \frac{A^7}{112}\right)y^7 + \text{etc.}$$

$$\therefore A=1, B=1/6, C=13/120, D=493/5040, E=37369/362880, \text{etc., in (5).}$$

$$x = \cos \theta = y + y^3/6 + 13y^5/120 + 493y^7/5040 + 37369y^9/362880 + \text{etc.,} \dots (A).$$

Substituting values, $y = 3\pi/20 = 0.471239 = \text{logarithm } 1.673241 +$.

$$2\text{nd} = 0.017441$$

$$3\text{rd} = 0.002517$$

$$4\text{th} = 0.000505$$

$$5\text{th} = 0.000118$$

$$\text{Estimated} = 0.000025$$

$$\cos \theta = 0.491845$$

$$\begin{array}{l} 2\text{nd term } y^3 = 1.019724 - \\ 6 \quad 0.778151. \end{array}$$

$$0.017441 = 2.241573$$

$$\begin{array}{l} 4\text{th term } y^7 = 3.712688 \\ 493/5040 \dots 2.990416 \end{array}$$

$$0.000505 = 4.703104$$

$$\begin{array}{l} 3\text{rd term } y^5 = 2.366206 \\ 13/120 = 1.034762 \end{array}$$

$$0.002517 + = 3.400968$$

$$\begin{array}{l} 5\text{th term } y^9 = 3.059171 \\ 37369/362880 \dots 1.012737 \end{array}$$

$$0.000118 = 4.071908$$

$$\text{Chord } AB = 10\sqrt{1 - \cos^2 \theta} = 8.7068 +. \quad ACD = 60^\circ 32' 17'' \text{ nearly.}$$

NOTE.—Formula (A) is also a general solution for the height of the circular segment (see problem 37, page 76, Vol. II). When the angle ACD is less than 50° , solve (1) for $\sin \theta$, and we have,

$$\theta < 50^\circ = \sin \theta = \left(\frac{3a\pi}{2d}\right)^{\frac{1}{3}} - \frac{1}{10}\left(\frac{3a\pi}{2d}\right)^{\frac{3}{5}} - \frac{1}{1400}\left(\frac{3a\pi}{2d}\right)^{\frac{5}{7}} - \frac{1}{22400}\left(\frac{3a\pi}{2d}\right)^{\frac{7}{9}} - \text{etc.,} \dots (B).$$

Chord $= D \cdot \sin \theta$. It will be noticed that the convergency, in part, depends on the smallness of the value of y .

PROBLEMS.

42. Proposed by E. B. ESCOTT, 6123 Ellis Avenue, Chicago, Illinois.

To find a triangle whose sides and median lines are commensurable.

43. Proposed by H. C. WILKES, Skull Run, West Virginia.

To find, if possible, a right angled triangle, the bisectors of the acute angles of which, can be expressed by integral whole numbers.